Abstract—This paper presents a novel collective coverage control strategy for robots to achieve effective coverage over a large-scale spill. The proposed idea is based on the divide and conquer approach that partitions a large irregular spill in the workspace into a number of smaller zones and let the robot team cover each packing zone sequentially. Ultimately, the robot team can cover the entire area. For an effective coverage operation in diverse and dynamic environments, we propose a pivot-based collective coverage control strategy performed by a pivot robot and multiple planet robots. The pivot robot is located at the center of the area to be covered and serves as a lighthouse to all the working robots denoted as planet robots exploring the area. By doing so, the planet robots do not require a global coordinate system nor massive communication between robots for their coordination, ultimately enabling an efficient, adaptive, scalable, and fault tolerant coverage operation. The proposed strategy is validated through extensive simulation experiments with different packing shapes and different numbers of robots.

Keywords—Coverage Control, Multi-Robot Coordination, Networked Robots, Artificial Potential Field.

I. INTRODUCTION

Coverage problems for mobile robots have been studied for a long time due to their wide and various applications such as oil spills cleaning [1]. However, many past researches have been limited to a single-robot scenario which is inferior in coverage efficiency and unconducive to large-scale situations [2]. Recently, coverage control using multi-robot systems to increase the coverage efficiency has been actively pursued [3]. However, high efficient, scalable, and fault tolerant coverage strategies for diverse and dynamic environments have not yet been fully discovered due to the challenges of coordination and communication among robots. Moreover, research is also underway to handle various workspace shapes and large-scale coverage problems, but the results are unsatisfactory.

In this paper, we propose a new collective operation control strategy with the advantages including high efficiency, strong scalability, fault tolerance, and unknown obstacle avoidance. This strategy first uses the divide and conquer approach and partitions a large-scale irregular spill into multiple small areas with certain basic geometric shapes, such as squares, triangles, hexagons, and circles. The partition process is also named as geometric packing or tessellation [4]. Then, using the proposed strategy, the robot team consisting of a pivot robot and multiple planet robots will cover each packing area sequentially and ultimately enables a fully coverage to the entire workspace. Fig. 1 illustrates the proposed collective control strategy. The pivot robot

![Fig. 1: An image that illustrates our proposed collective coverage strategy for various geometric packing shapes, including circles, squares, triangles, and hexagons for irregular spill coverage. Blue dashline shows the connectivities between planet robots (yellow) and the pivot robot (green). Red dashline represents one robot being in the vision of another robot. (Photo credit to Smithsonian.com)](image-url)
robots start performing coverage after the pivot robot is designated position under operator’s guidance. The planet that the centroid of the packing area should reside in is shown as $\Delta C$, and has a smooth boundary; the spill is featured in the range of wireless connectivity, i.e., $r_{\text{vision}}$ and $r_{\text{comm}}$ denote the vision sensor and wireless connection ranges for planet robots. The purple dots indicate the deployment goals for planet robots. The three cases are demonstrated by $\text{Situation I}$, $\text{Situation II}$, and $\text{Situation III}$ in Fig. 2a.

Meanwhile, the radius of the packing area cannot exceed the robot’s ability to patrol such an area, $r_{\text{PV}}$, and $r_{\text{comm}}$, respectively. If $\text{Situation I}$, the control algorithm should enable robots to move around and patrol the spill for the robot will have enough space to travel around and clean the spill.

We use $\partial P_i$ and $\partial S_i$ to denote the boundary of $P_i$ and $S_i$, respectively. If $\partial S_i$ does not exist or is not closed initially since $S_i \notin P_i$, the control algorithm should ensure that robots can travel uniformly in a circular manner. In this case, robots can track the spill boundary $\partial S_i$ and realize complete coverage over $S_i$.

We further state the following assumptions that our research is based on: The spill to be covered is overall integrated and has a smooth boundary; the spill is featured by colors or textures that can be distinguished from the base using image processing techniques.

B. Pivot-based Coverage Control Strategy

The coverage operation starts when the packing process is completed. The targeted large-scale spill is partitioned into $M$ number of small packing areas with specific shapes, note that the centroid of the packing area should reside in $S_i$. Meanwhile, the radius of the packing area cannot exceed the robot’s ability to patrol such an area, $r_{\text{PV}}$, and $r_{\text{comm}}$, respectively. The planet robots may not be able to identify the pivot robot and localize itself. We first deploy the pivot robot to the centroid of the first packing area $S_1$. The pivot robot can be randomly selected and it can move autonomously to the designated position under operator’s guidance. The planet robots start performing coverage after the pivot robot is ready. The connectivity can be built between the planet robot to the pivot robot by simply using a paired Wi-Fi Access Point (AP) (for pivot robot) and an adapter combined with an antenna (for planet robot). We can localize the planet robot, including bearing and distance, through Wi-Fi signal strength of the pivot robot [5], [8].

We show the coverage model here. When a robot is maneuvering in $S_i$ in $\text{Situation I}$ and traveled points $A(x_1, y_1)$ and $B(x_2, y_2)$ in sequence, the coverage manifold can be formulated as:

$$\Delta C = \{M(x, y) : (0 < AM \cdot AB < AB \cdot AB) \cap (0 < AM \cdot AD < AD \cdot AD)\}$$

(3)

$$D(x_4, y_4) = \left( -\sqrt{||AD||^2 - y_4^2} , \frac{||AB||^2 + ||AD||^2 - ||BD||^2}{2||AD||} \right)$$

(4)

where $M$ is a point in $\Delta C$, and $||AD|| = d$ is the horizontal coverage distance. The coverage manifold is demonstrated in Fig. 2a. Furthermore, assuming a maximal processing capacity $\mathcal{V}$ of a robot in removing the spill, such as the centrifugal volume to algae spill harvesting and the filtering system capacity to oil spill cleaning, by having $\Delta C = vd\Delta t = \mathcal{V}\Delta t$, the maximum speed of the robot is bounded by $q_{\text{max}} = \frac{\mathcal{V}}{d}$.

The coverage model (3) can be interpreted as the robot cleans the spill $d$ distance to its left during maneuvering. This model can be applied in existing robots such as [9]. Note $d$ value relies on the curvature of the trajectory, and can be approximated based on the contour of the spill. We choose left hand side coverage for the purpose that we want all the robots to travel uniformly counterclockwise to avoid the collision and realize a distributed coverage behavior. The details will be elaborated later.

Now we provide the coverage control algorithm that enables a full coverage to the packing area to remove the spill, and converges to the pivot when the operation reaches an end. Using $q_i = (x_i, y_i)$ and $q_{pv} = (x_{pv}, y_{pv})$ to denote the position of a planet robot $R_i$ and the pivot robot $R_{pv}$, and $r_{pj}$ to denote the radius of packing area $P_j$, Algorithm 1 shows the procedure of the coverage control that runs iteratively until the operation is completed.

To illustrate Algorithm 1, we present a finite state transition diagram consisting of four states in Fig. 3. Meanwhile, we sketch two possible initial situations under circular packing with randomly distributed robots in Fig. 2b. In the transition diagram, every robot starts with deployment (State 1), where it moves toward the goal position on $\partial P_j$. Especially, if the robot experiences a traverse $S_j \rightarrow \sim S_j$, it stops at the traverse point on $\partial S_j$ and terminates deployment. This case is demonstrated by $R_6$ in Situation II of Fig. 2b. State 2 means if the robot is deployed but does not detect $\partial S_j$ or reside in $S_j$, it will start moving toward the pivot robot until it detects $\partial S_j$. This state is demonstrated by $R_5$ in Situation II. Note that the robot can travel as fast as it can in State 1 and 2, since it is not performing the coverage. In State 3, if no $\partial S_j$ is detected but the robot resides in the spill, it will maneuver along $\partial P_j$ and create a boundary $\partial S_j$...
that trailing robots can follow in State 4. All the robots in Situation I and robots $R_1$, $R_3$, $R_4$, and $R_N$ in Situation II demonstrate State 3, while $R_2$ and $R_6$ in Situation II demonstrate State 4. The coverage of this packing area ends up with all planet robots gathering around the pivot robot within a threshold $\varepsilon$ according to the robot dimensions.

C. Motion Control based on Artificial Potential Field

To motivate robots to track $\partial S_j$ or $\partial P_i$ in order to cover the spill collectively but avoid collision, we introduce artificial potential field based motion controllers for each state. To avoid local minima and deadlock, we propose a pre-prioritized collision avoidance strategy to coordinate the robots working in different states. Meanwhile, all the robots performing coverage over the spill shall move counterclockwise (CCW) about the pivot robot when tracking $\partial S_j$ or $\partial P_i$. A uniform moving direction leads to a distributed coordinate control.

The proposed avoidance priority is indicated in Fig. 3, the robots working in lower priority have to yield to those working in higher priority, and robots performing the coverage have the highest priority. The avoidance priority for these states are determined as $3 = 4 > 1 = 2$. No transition exists between State 2 and State 3, because the robot converges to the pivot only if nothing is detected. Furthermore, special consideration should be given to the robots working in the states of the same priority. As we stated above, robots working in State 3 and 4 (i.e., tracking either $\partial S_j$ or $\partial P_i$) are moving uniformly CCW. Given a continuous and smooth spill boundary consisting of $\partial S_j$ and $\partial P_i$, the collision happens only between a leading robot and its trailing robot.

Algorithm 1: Collective Coverage Control Algorithm for every planet robot in packing area $P_j$

```
repeat
    for $R_i$ under deployment (State 1) do
        $R_i$ moves from $q_i$ to $q_{d,i} + \frac{q_{i} - q_{pv}}{||q_{i} - q_{pv}||}r_{P_j}$;
        if $R_i$ experiences a traverse $\partial S_j \rightarrow -S_j$ then
            $R_i$ stops at $\partial S_j$;
    for $R_i$ under coverage operation do
        if No $\partial S_j$ is detected within $r_{vision}$ then
            if $R_i$ resides in $S_j$ then
                $R_i$ covers the spill under (3) by tracking $\partial P_j$ and hence creates $\partial S_j$ (State 3);
            else
                $R_i$ moves toward $q_{pv}$ (State 2);
        else if $\partial S_j$ is detected within $r_{vision}$ then
            $R_i$ covers the spill under (3) by tracking $\partial S_j$ (State 4);
            if $||q_i - q_{pv}|| \geq r_{P_j}$ then
                $R_i$ switches to track $\partial P_j$ and continues the coverage operation (State 3);
            else
                $R_i$ fails in vision sensing, it stops and becomes an obstacle;
                Bounding speed $\dot{q}_i = \frac{\varepsilon_i}{d}$;
        until $||q_i - q_{pv}|| \leq \varepsilon, \forall i \in \{1, ..., N\}$;
```

However, the robot working in State 3 does not have a leading or trailing robot, hence, we will limit the possible collisions to be from the robots working in State 4.

Similarly, for the robots working in State 1 and 2, since they are moving either towards or against the pivot robot, no collision is foreseen if their trajectories are non-coincident. Such a pre-prioritized collision avoidance rule eliminates local minima which halting the robot, and relies only on local sensing. In terms of deadlock, although not observed in the validation experiments, it can be tackled with appropriate motion planning methods [10]. In practice, robots can recognize each other’s states with minimum communication or via light signals and near field communication such as Radio Frequency Identification (RFID) [11].

Assuming a single integrator model for planet robot control, i.e.,

$$q_i(t) = u_i(t), \quad i \in \{1, ..., N\}$$

where $u_i(t) \in \mathbb{R}^2$ denotes the control input for planet robot $R_i$ at time instant $t$, the detailed motion control law for every state is provided below from Sec. II-C.1 to Sec. II-C.4.

1) Motion Controller for State 1: For robots working in State 1, provided the goal positions for deployment $q_i = q_{d,i} + \frac{q_i - q_{pv}}{||q_i - q_{pv}||}r_{P_j}$, we construct an attractive potential field formulated as

$$U_d^{(1)}(q_i) = \frac{1}{2} \xi_1 ||d(q_i, q_{d,i})||_2^2$$

(6)

where $\xi_1$ is a scaling parameter and $||d(q_i, q_{d,i})||_2$ is the distance between the current positions of robots and their goals obtained from radio signal strength measurement [5].

Additionally, we propose a repulsive potential exerted on the robots to avoid the collision. Such repulsive potential field is formulated as

$$U_r^{(1)}(q_i) = \frac{\xi_2}{2} \left( \frac{-1}{||d(q_i, q_{d,i})||_2} \right)^2, \quad \text{if} \quad ||d(q_i, q_{d,i})||_2 \leq d_0,$$

(7)

$$0, \quad \text{otherwise},$$

where $\xi_2$ is a positive scaling factor, $||d(q_i, q_{d,i})||_2$ denotes the distance between robot $R_i$ that works in the states of higher or equal avoidance priority to other robots $q_i$ that within the
effective range, which values are determined by the robot dimensions.

Due to this, for any planet robot, the final constructed potential field for motion control is:

\[ U_f^{(1)}(q) = U_d^{(1)}(q) + \sum_{i \in N_{\text{vision}}^{(3,4)}} U_i^{(1)}(q) \]  

(8)

where \( N_{\text{vision}}^{(3,4)} \) represents robots working under State 3 and 4 and within the vision sensing range \( r_{\text{vision}} \).

With (8), the control input for the robot working in State 1 is obtained:

\[ u^{(1)} = -\nabla U_f^{(1)}(q) = \left\{ \begin{array}{l}
\xi_1(q_g - q) + \\
n_{\text{vision}}^{(3,4)} \sum_{i \in N_{\text{vision}}^{(3,4)}} \left( \frac{1}{d_0} - \frac{1}{\|d(q, q_i)\|^2} \right) \xi_2 \nabla d(q, q_i), \\
\xi_1(q_g - q),
\end{array} \right. \]

(9)

where \( u = [u_1, u_2, ..., u_n] \) is the input velocity of all the robots under deployment. Moreover, the input velocity \( \|u_i\| \) is bounded by a maximum value according to Algorithm 1.

2) Motion Controller for State 2: Similar to State 1, the source of attractive potential becomes the pivot robot \( R_{pv} \), thus, we have attractive potential field formulated as

\[ U_d^{(2)}(q) = \frac{1}{2} \xi_3 \|d(q, q_{pv})\|^2 \]

(10)

where \( \xi_3 \) is a scaling parameter and \( \|d(q, q_{pv})\| \) the distance between robot current positions and pivot robot.

According to the pre-decided priority for collision avoidance, the repulsive potential field for robots working in State 2 is constructed the same as State 1, which results in a control law shown in (12):

\[ U_f^{(2)}(q) = U_d^{(2)}(q) + \sum_{i \in N_{\text{vision}}^{(3,4)}} U_i^{(2)}(q) \]

(11)

\[ u^{(2)} = -\nabla U_f^{(2)}(q) = \left\{ \begin{array}{l}
\xi_3(q_{pv} - q) + \\
n_{\text{vision}}^{(3,4)} \sum_{i \in N_{\text{vision}}^{(3,4)}} \left( \frac{1}{d_0} - \frac{1}{\|d(q, q_i)\|^2} \right) \xi_2 \nabla d(q, q_i), \\
\xi_3(q_{pv} - q),
\end{array} \right. \]

(12)

3) Motion Controller for State 3: For the robot moving along \( \partial P \) in State 3, since it has no leading or trailing robot in the same state, it requires only attractive potential to motivate movement. In order to avoid a discrete control method which typically results in frequent position updates and improve control efficiency, we propose a continuous control law (13) and enable robots moving along \( \partial P \).

We use attractive potential (10) to motivate the movement of the robot. However, different from State 2, the robot in State 3 has to move in a tangent way along \( \partial P \) and CCW about the pivot. Thus, we determine the control law as below:

\[ u^{(3)} = -T \cdot \nabla U_d^{(1)}(q) = T \cdot \xi_3(q_{pv} - q), \]

\[ T = \begin{bmatrix}
\cos(-\pi/2) & -\sin(-\pi/2) \\
\sin(-\pi/2) & \cos(-\pi/2)
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-1 & 0
\end{bmatrix}. \]

(13)

Particularly, it is difficult to keep the robot in the orbit and tracking \( \partial P \) with only attractive potential, as a disturbance may yield \( \|q_{pv} - q\| > r_{\text{comm}} \) and break the robot connection to the pivot robot. Due to this, we can introduce an asymmetric potential function such as (26) in [12] and let \( \rho_2 = r_{\text{comm}} \) and \( \rho_0 = r_{\text{pj}} \) for a stabilized motion.

4) Motion Controller for State 4: Since collision happens only between leading robot and its trailing robot if they are working in State 4, we construct an attractive potential \( U_d^{(4)} \) exerted on robot \( R_i \) by its leading robot \( R_{i+1} \). When \( R_i \) is tracking \( \partial S \), the measurement to the length along the spill boundary between itself and other agents within its vision sensing range is used to construct the potential field. Practically, such measurement can be performed with stereo vision sensors, LIDAR, or high resolution laser rangefinders, along with the techniques developed in [13], [14]. Suppose there is a function \( s = f(q) \) to represent the spill boundary \( \partial S \), where \( s \in [0, \|\partial S\|] \) indicates the length between a reference point and an agent of position \( q \) in CCW. We can show that once a robot \( R_{i+1} \) falls within the vision range of its trailing robot \( R_i \), the distance between the two neighboring robots, namely \( l_i = \|q_i, q_{i+1}\| \), can be decided by

\[ l_i = \begin{cases} 
s_{i+1} - s_i, & \text{if } s_{i+1} \geq s_i, \\
 s_i - s_{i+1} + \|\partial S\|, & \text{if } s_{i+1} < s_i.
\end{cases} \]

(14)

If the leading robot of \( R_i \) is beyond its vision range \( r_{\text{vision}} \), we define a virtual distance \( l^*_i \) and update (14) as below:

\[ l^*_i = \begin{cases} 
l_i, & \text{if } l_i \leq r_{\text{vision}}, \\
l_{\text{vision}}, & \text{if } l_i > r_{\text{vision}} \text{ or } l_i \text{ is unknown.}
\end{cases} \]

(15)

The attractive potential function is then defined as

\[ U_d^{(4)} = \frac{1}{2} \xi_4 l^*_{i}^2 \]

(16)

where \( \xi_4 \) is a positive scaling factor. The linear velocity input \( u_i \) should be in the direction of the negative gradient of \( U_d^{(4)} \) with respect to \( s_i \), such that

\[ u_i^{(4)} = -\nabla s_i U_d^{(4)} = \xi_4 l^*_i. \]

(17)

It can be easily proved that the attractive potential \( U_d^{(4)} \) can motivate robot \( R_i \) to move forward and as well prevent collision with its leading robot, since it is always non-negative and \( u_i \to 0 \) if \( l^*_i \to 0 \). Furthermore, all the planet robots will gather and stop around pivot robot when the coverage control is completed.
D. Coverage to the Next Packing Area

When a group of robots completes the coverage to the current packing area \( P_i \) and removes the spill \( S_i \), the pivot robot, after relocation by human operator teleoperation or navigation, can herd all the planet robots into the next packing area \( P_{i+1} \) to perform a new round of coverage operation. One of the practical herding strategies can be found in [15], which features robot connectivity preservation.

III. Simulation Experiments

Extensive scaled-down simulation experiments were conducted to validate the proposed collective coverage strategy in Robotarium platform [16], which is a MATLAB based open source multi-robot coordination toolbox. Robotarium has a 2D arena of size \( 3m \times 3m \), where disperse scaled-down robots are deployed to simulate the collective coverage operation. A variety of scenarios were designed and tested for validation. In the experiments, \( R_1 \) denotes the pivot robot and is located at the center of the arena. The linear velocity of the robot is bounded by \( v_{\text{max}} = 0.075 \text{ m/s} \). The average width \( d \) in Fig. 2a is set to be 0.33 m.

A. Evaluation Metrics

The following metrics are decided for our experiments to evaluate the performance of the proposed solution:

1) Lyapunov candidate function (Convergence):

\[
L = \sum_{R_i \in P, j \neq i} \| q_i - q_j \|, \quad \text{or} \quad L = \sum_{R_i \in P} \| q_i - q_p \|. \tag{18}
\]

The second function is used if only one planet robot is involved, where \( q_p \) is the location of the pivot robot.

2) The number of iterations \((k_{\text{stop}} \leq k_{\text{max}})\) to reach the following stop condition:

\[
\text{Stop at } k_{\text{stop}} \text{ if the current area } A(t) \leq A_{\min}. \tag{19}
\]

Here \( A_{\min} \) is defined to be 1% of the initial area.

B. Experiment Scenarios

The following four scenarios are designed to demonstrate the efficacy and efficiency of the proposed solution:

- Sc. 1 - Adaptiveness to various packing shapes
- Sc. 2 - Scalability which allows multiple robots
- Sc. 3 - Fault tolerance to the robot with coverage failures
- Sc. 4 - Unknown obstacle during the operation
- Sc. 5 - Sequential coverage to multiple packing areas

All the scenarios demonstrate the convergence of the planet robots at the end of the coverage operation. The experiment videos are available at https://goo.gl/K6u589.

C. Scenario 1 - Adaptiveness to Various Packing Shapes

To demonstrate the adaptiveness to different packing shapes with our proposed collective coverage control strategy, we select four representative geometric shapes including triangle, square, hexagon, and circle. In contrast, [17] and [1] can deal with only square cell coverage.

Among all the four shapes, the circular packing method can maximize the wireless communication range, i.e., can have possibly a greater packing area with less travel of robots than other shapes. However, as the main disadvantage, circular packing allows 90.69% coverage rate with unique radius circles and hexagonal lattice arrangement [18]. To improve the coverage rate, one can use circles with different radii and increase the packing densities to be > 91% [19]. Triangular, square, and hexagonal packing methods can achieve 100% coverage rate; however, the packing shapes have to be aligned adequately to avoid overlap or gap. In practice, the alignment issue can be circumvented by assuming a uniform orientation for all packing areas using magnetic compasses. Note that \( r_{P_j} \) in Algorithm 1 is not a constant for non-circular packing methods, hence (13) may not apply. Nevertheless, \( r_{P_j} \) can be obtained by referencing the attitude of the pivot robot and using estimation methods such as [20].

The coverage performance and robot trajectories of the four packing shapes are shown in Fig. 4. One can see that a complete coverage over these areas is achieved with a eight-robot team working under our proposed control strategy. Thus, any large-scale spill which can be partitioned into several packing areas shall can be covered in the same manner. Particularly, we present area evolution snapshots for the circular packing coverage in Fig. 5, which demonstrate the evolution of robot trajectories and depict the remaining area with shadow.

Fig. 6a and 6b show the coverage performance under Situation II of Fig. 2b, where \( \partial S \) already exists at the beginning of the operation. In this case, robots \( R_5 \) and \( R_6 \) detected \( \partial S \) when continuously heading toward the pivot robot, then they entered State 4 and started following \( \partial S \), rather than moving along \( \partial P \) as the other robots.
Fig. 6: (a) Initial distribution of robots demonstrating Situation II of Fig. 2b, where ∂S already exists before task. (b) The trajectories of robots. (c) An abrupt obstacle was introduced during the operation in time \( t = 1300 \). (d) Trajectories of the robots that succeeded in avoiding the obstacle and performed complete coverage.

Fig. 7: The first row shows the initial random distributions of robots; the darker disk is circular packing area in the workspace. The pivot robot is 1 and located at the centroid of the area. The second row shows the robot trajectories, with the amount of robots being 2, 6, 11, and 16, respectively.

D. Scenario 2 - Scalability Which Allows Multiple Robots

To improve the efficiency of coverage, we further demonstrate the scalability of our proposed solution that allows any number of planet robots. We take circular packing as an example, one can easily apply the same control scheme to other packing methods.

We start from the minimal number of robots \( N = 2 \), with one of them serving as pivot robot while the others being planet robots. We scale up \( N \) with the same interval of five and therefore obtain four scenarios with \( N = \{2, 6, 11, \text{ and } 16\} \). The initial distributions and trajectories of the robots during operation are shown in Fig. 7.

The area evolution during the cleaning process is shown in Fig. 9a, from which we conclude the improvement in the efficiency of our proposed solution. The area of the cleaning zone decreased steadily as time elapsed. The \( k_{\text{stop}} \) was reduced significantly while having more robots deployed in the cleaning operation. Nevertheless, more robots employed does not always lead to a shorter operation time. Time may be consumed in robots avoiding each other in a crowd. The results in Fig. 7 also validate the effectiveness of our robot motion controller and collision avoidance strategy by showing a smooth trajectory for every robot. The \( k_{\text{stop}} \) values for all the scenarios are listed in Table I.

At the end of the operation, all the robots gathered around the pivot robot as indicated in Fig. 7. This characteristic potentially allows further deployment such as moving to the next packing area or docking for recharging. Such convergence property is validated through Lyapunov candidate function (18) and is demonstrated in Fig. 9b. In particular, convergence in scenario \( N = 2 \) was step-like during operation, because the second Lyapunov candidate function in (18) was used for evaluation. While the planet robot was covering \( \mathcal{P} \), it approached the pivot in an intermittent way. Additionally, due to physical restraint of robots, the Lyapunov candidate function value did not converge to zero when the task was completed.

We also consider the energy consumption during the collective cleaning task, which can be reflected by the entire traveling distance of robots employed. The entire traveling distances with different \( N \) are indicated in Table I. Noticeably, at the expense of a shorter operation time, the total traveled distance increased when more robots were deployed. The evolution of traveled distance is shown in Fig. 9c.

E. Scenario 3 - Fault Tolerance to the Robot with Coverage Failures

We demonstrate fault tolerance capability of our proposed solution when the robot has coverage failure. Once employed in operation, a robot may face many issues that prevent it from removing the spill, e.g., a chemical substance removal robot facing filtering system failures, or an algae harvesting robot being fully loaded with algae. Even if the faulty robots can be simply isolated from the operation, we still want to let them converge to the pivot robot and collect them at the end of the operation, if they still have mobility.

In our experiment, the robots have the same circular packing setting demonstrated in Fig. 4. However, \( R_4 \) lost its coverage ability at \( t = 1500 \) but regained it at \( t = 1500 \). Fig. 9d shows \( R_4 \) kept moving along \( \partial S \) and converge to the pivot robot when encountered such coverage failure. Due to this, the spill area evolution shown in Fig. 9e as the red dashline indicates a falling in the spill removal rate at \( t = 500 \), but it witnesses an increase at \( t = 1500 \) because \( R_4 \) regained the coverage ability.

F. Scenario 4 - Unknown Obstacle During the Operation

Unavoidably, the robots may encounter obstacles during the coverage operation in an unknown environment, such as rocks in water areas and robots that lose mobility. We demonstrate the capability of our strategy in abrupt obstacle

![Fig. 8: The coverage evolution of a large-scale spill partially shown with snapshots using four robots after circular packing partition.](image-url)
avoidance. As shown in Fig. 6c, an abrupt obstacle was introduced in time $t = 1300$; however, the robot team succeeded in avoiding this obstacle using approaching sensors such as sonars and LIDARs, and achieved complete coverage over the area still, as shown in Fig. 6d.

G. Scenario 5 - Sequential Coverage to Multiple Areas

We demonstrate a large spill coverage scenario using four robots after partition with circular packing as described in Sec. II-D. The large spill was removed in sequence by covering every individual packing area, which process is partially depicted with snapshots in Fig. 8 due to space limit.

IV. CONCLUSION AND FUTURE WORK

In this paper, a novel pivot-based collective coverage strategy with a multi-robot team is proposed for large-scale spills coverage problems. The proposed strategy is validated through simulation experiments to be adaptive to various packing areas and can achieve a complete coverage, and further realizes the cleanup to the large-scale spill by covering all the partitioned packing areas in sequence. With the proposed solution, all planet robots will converge to the pivot robot at the end of the operation, which enables further operations such as cleaning up the next partition area, docking for recharging, etc. This collective coverage strategy features strong capabilities including scalability, fault tolerance to coverage failures, and abrupt obstacle avoidance, which further enhance its efficiency and robustness. Our future goals mainly focus on validating our solution with field tests, and promoting collaboration among multiple teams.

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